Spatially-Explicit Integrated Uncertainty and Sensitivity Analysis of Criteria Weights in Multicriteria Land Suitability Evaluation

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SOFTWARE AVAILABILITY

Name of the software: Integrated Uncertainty and Sensitivity Analysis (iUSA), v1

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Software required: Python 2.7, NumPy 1.6

Available since: May 2014

Available from: http://www.geo.msu.edu/~stsa/ including documentation and demonstration data

Program language: Python

Cost: Free

Package size: ca. 32 Kb excluding documentation and demonstration data

ABSTRACT

We employ spatially-explicit uncertainty and sensitivity analysis to examine the robustness of land suitability evaluation. We use Monte Carlo simulation to sweep through criteria weight space, where weights are expressed using probability distributions. Multiple output suitability maps are generated and summarized using: an average suitability map, a standard deviation uncertainty map, and a number of sensitivity maps. We demonstrate how these surfaces help detect critical regions of suitability on the example of habitat suitability evaluation for a wetland plant. Areas of high average suitability and low uncertainty signify robust suitability sites, whereas high average suitability and high uncertainty characterize candidate areas. These candidate areas are potentially suitable but need further examination with variance-based sensitivity analysis, in which the variability of land suitability is decomposed and attributed to individual criteria weights. The resulting sensitivity maps delineate regions of weight dominance, where a particular weight greatly influences the uncertainty of suitability scores.

1. Introduction

Multicriteria land suitability evaluation (LSE) is one of the best known application domains of spatial multicriteria evaluation – S-MCE (Jankowski, 1995, Carver, 1991, Malczewski, 2006, Pereira and Duckstein, 1993, Eastman et al., 1995, Chakhar and Mousseau, 2008, Densham, 1991, Malczewski, 2004, Chen and Paydar, 2012, Yu et al., 2011, Chen et al., 2010a). The procedure, aimed at evaluating the potential of a given location for a particular land use, involves a set of quantifiable spatial criteria, their standardization functions, techniques for expressing preferences regarding the relative importance of the criteria, and aggregation rules combining quantified criterion preferences with standardized criterion values into an overall suitability score (Malczewski, 2004, Lodwick et al., 1990). The score is then assigned to each land unit and may be used as the bases for land use allocation. The result of the procedure is a land suitability map.

An important step in the S-MCE procedure, and at the same time a source of uncertainty, is the articulation of preferences in regard to spatial criteria. The uneven importance of criteria may result from policies, established hierarchies, cause-effect relationships, and subjective preferences. Criteria preferences are commonly quantified as a set of weights. There are various techniques of eliciting criteria preferences and transforming them into numeric weights (Malczewski 1999). Simple techniques such as *ranking* or *rating* require merely arranging criteria in a monotonic rank-order and feeding the ranks into a formula, which then yields a weight for each criterion. More complex and theoretically founded technique called *pairwise comparison* derives weights from a series of criterion-to-criterion comparisons, based on the 1-9 interval scale, and processes the comparison scores using linear algebra operations. A different approach to calculating criteria weights is based on examining trade-offs between values attained by one criterion versus another. All of these techniques involve judgment and are open to cognitive limitations of human information processing, thus contributing to uncertainty inherent in any S-MCE procedure.

Much progress has been made over the last twenty years in developing methods of multicriteria land suitability evaluation, especially in integrating GIS with S-MCE (Pereira and Duckstein, 1993, Joerin et al., 2001, Chakhar and Mousseau, 2008, Malczewski, 2006, Jankowski, 1995, Dragan et al., 2003, Larson and Sengupta, 2004, Chen et al., 2001). One methodological area of LSE receiving less attention has been spatially-explicit, integrated uncertainty and sensitivity analysis (iUSA) as a systematic approach to accounting for the inherent uncertainty of the S-MCE process (Chen et al., 2011, Ligmann-Zielinska and Jankowski, 2008, Gómez-Delgado and Bosque-Sendra, 2004, Lilburne and Tarantola, 2009, Crosetto et al., 2000, Benke and Pelizaro, 2010). S-MCE follows Simon's normative model of decision making process comprised of three phases: intelligence-design-choice (Simon, 1957). In this model, uncertainty and sensitivity analysis is the core of the final 'choice' stage, where one evaluates decision alternatives and recommends one of them as a decision problem solution. In this context, the uncertainty involved in selecting a decision alternative arises from the character of data, ignorance of system drivers, diversity of human values, or whimsical preferences to name just a few. Specifically, uncertainty in S-MCE may be concealed in model inputs including the selection of decision criteria that reflect the objective of the analysis, criteria measurement (attribute uncertainty, positional uncertainty, and measurement errors), and preferences (weights) associated with the decision criteria. Consequently, models should be thoroughly evaluated to ensure their robustness under a wide range of possible input conditions, where robustness is defined as a minimal response of model outcome to changing inputs (Ligmann-Zielinska and Jankowski, 2008, Gómez-Delgado and Bosque-Sendra, 2004).

Conceptually, iUSA in LSE can be defined as a systematic approach to quantifying the variability of outcomes of multicriteria evaluation given model input uncertainty (uncertainty analysis - UA), and then identifying, which inputs (e.g. decision criteria or criteria weights) are most responsible for this variability (sensitivity analysis - SA). A well-structured and thorough uncertainty and sensitivity analysis leads to identification of inputs that need more attention (by, for example, better calibration of model parameters), and to model simplification by discarding inputs that have little impact on outcome uncertainty (Saltelli and Annoni, 2010, Lilburne and Tarantola, 2009, Saltelli et al., 2008). It is also one of the required steps in LSE-based adaptive management (Labiosa et al., 2013).

Several approaches to uncertainty and sensitivity analysis in LSE have been proposed. They involve a systematic examination of S-MCE model components by changing the set of criteria, criteria weights, or aggregation functions, and rerunning the model for each change in model components and their values (Chen et al., 2010b, Ligmann-Zielinska and Jankowski, 2008, Gómez-Delgado and Bosque-Sendra, 2004, Chen et al., 2013). Additionally, for highly uncertain criteria, a modeler can introduce error by adding a randomly generated uncertainty surface, which is used to produce an alternative model output (Krivoruchko and Gotway Crawford, 2005). These approaches lead to generating several composite scores, which can be compared visually and statistically to assess the uncertainty associated with each decision alternative.

The simplest and the most straight-forward method of uncertainty and sensitivity analysis is to change one decision component at a time (while keeping all other factors constant) and observe the resulting changes in model output. This approach, called One-Factor-At-A-Time (OAT) (Daniel, 1958), is probably the most popular among spatial and other environmental modelers. Chen et al. (2010b) and Chen et al. (2013) present examples of OAT to explore the sensitivity of irrigated cropland suitability to changes in criteria weights. Rae at al. (2007) use three variations of base-case inputs (land cover categorization, spatial resolution, and surface error) to produce alternate reserve designs. In their habitat suitability analysis of an old-forest polypore, Store and Kangas (2001) use different weights for the two major decision criteria (vegetation and soil characteristics) to produce three alternative suitability maps and demonstrate how changes in importance assigned to habitat characteristics affect the decision to select areas of vegetation preservation.

The popularity of OAT can be attributed to a number of reasons. First, it is a very intuitive approach to track shifts in model outcome. An analyst can change a particular input value by some percentage and assess whether the change in the output is of similar magnitude. For example, if the changes in suitability scores are less than the introduced change in criterion weight, the analyst may assume that the model is not very sensitive to this weight (Triantaphyllou and Sánchez, 1997, Malczewski, 1999). Second, OAT lends itself well to visual exploratory spatial data analysis, where a decision maker can interactively manipulate inputs (such as criteria weights) and instantaneously observe the results (Chen et al., 2010b, Chen et al., 2013, Jankowski et al., 2001, Jankowski et al., 1997). Third, due to its simplicity, OAT does not require any prior knowledge about model sensitivity analysis (Chen et al., 2013). Fourth, OAT is very efficient computationally as it does not require a large number of model executions. As noted by Saltelli and Annoni (2010), however, the OAT approach has some serious limitations. First, conventional OAT is commonly performed using an interactive mode, which on the one hand promotes exploration, but on the other hand opens the door to arbitrary change of parameter (e.g. criterion weight) values. In the context of LSE, it may be hard to decide, which criterion weight to change and by what amount. There is no guarantee that the magnitude of value change accurately reflects the true range of impact of this criterion on the outcome variability. Second, OAT does not account for the magnitude of the overall impact of input uncertainty on the resulting model output. Finally, OAT does not take into account potential interactions occurring among model components. Specifically, it does not provide any quantitative information on the joint contribution of a selected input, interacting with other inputs, to the total unconditional output variability. The latter limitation of OAT is particularly troubling in spatially heterogeneous problems, where inputs can be spatially autocorrelated or can locally co-vary.

Variance-based global sensitivity analysis (GSA) has been proposed as an alternative to OAT (Homma and Saltelli, 1996, Saltelli et al., 1999, Saltelli et al., 2008) . The objective of variance-based GSA is to partition the variability of model outcomes and apportion the fractions to inputs in order to obtain quantitative measures of input influence on output uncertainty. As a result, two sensitivity indices are computed for each input element of the model; a first order index (S) that captures the independent contribution of a given input on output variability, and a total effect index (ST) that also accounts for interactions among a given input and other inputs. The (S,ST) pair offers a succinct yet comprehensive measure of input influence that does not depend on model formulation. Since variance-based sensitivity analysis belongs to global methods (Saltelli et al., 2008), it provides means of studying the whole range of input conditions rather than the best-guess or ad-hoc selected values, as it happens in the OAT approach to sensitivity analysis.

Variance-based GSA has already been introduced in GIS-based modeling, although it has not yet gained a widespread acceptance. Examples of GIS applications of the variance-based GSA include hydrologic modeling aimed at flood forecasting (Crosetto and Tarantola, 2001, Crosetto et al., 2000), planning hazardous waste disposal (Gómez-Delgado and Tarantola, 2006), simulating groundwater flow and contamination (Lilburne and Tarantola, 2009, Saint-Geours and Lilburne, 2010), modeling land use (Ligmann-Zielinska and Jankowski, 2010, Plata-Rocha et al., 2012), and hydro-geological modeling (Marrel et al., 2011).

Regardless of the specific method of sensitivity analysis used, we argue that the most essential requirement of a comprehensive uncertainty and sensitivity analysis of the LSE model and its results is that it should be performed in a spatially-explicit manner. Given the spatial nature of LSE modeling, where spatial input layers directly contribute to generating spatial output suitability maps, the result of uncertainty and sensitivity analysis should also be represented in spatial format. The importance of spatially-explicit uncertainty and sensitivity analysis in S-MCE was first noted by Herwijnen and Rietveld (1999). Tarantola et.al. (2002) used spatially dependent input and variance-based global sensitivity analysis to augment environmental decision analysis, but their study did not account for spatial variability of the sensitivities. Feick and Hall (2004) made the first operational attempt to map weight sensitivities (represented as sensitivity indices) within the S-MCE context. Rinner and Heppleston (2006) proposed another method of post-hoc adjustment of suitability maps to account for the effect of spatial variability in criteria values. While Gómez-Delgado and Tarantola (2006) used spatially-inexplicit variance-based GSA to calculate sensitivity indices of various S-MCE inputs to evaluate the robustness of landfill location, Chen et al. (2010b) and Chen et al. (2011) used spatially-explicit OAT to generate uncertainty and sensitivity analysis results in the form of alternative suitability or rank maps. Malczewski (2011) introduced an explicit method of capturing spatial variability by means of a local multicriteria aggregation procedure. Plata-Rocha et al. (2012) applied error surfaces in extended-FAST global SA to introduce weight variability across space and to evaluate positional accuracy of criteria used to calculate land use suitability. Finally, Ligmann-Zielinska and Jankowski (2012) offered a procedure for adjusting criteria preferences based on a proximity relationship.

This paper contributes to methods of spatially-explicit uncertainty and sensitivity analysis in S-MCE in two ways; first, by addressing the problem of high suitability areas burdened by high uncertainty of S-MCE results, and second, by presenting an approach for identifying specific S-MCE inputs contributing to high uncertainty. The approach is based on the premise that, in problems characterized by spatial heterogeneity of solutions, iUSA should be explicitly preformed for each and every spatial unit that comprises the study area. The proposed approach is based on Monte Carlo simulation that, in the context of LSE, results in producing multiple suitability surfaces. The aggregate of the surface is then obtained by computing the average (mean) suitability surface and the standard deviation surface, which provides a measure of uncertainty present in the average suitability surface. The standard deviation (uncertainty) surface is then used to calculate multiple sensitivity maps. Consequently, the method quantifies and maps the uncertainty of land suitability and the sensitivities of LSE model inputs for every spatial unit within the area of interest. The novelty of the presented approach comes with two maps of sensitivity indices for every model input: an S-map and an ST-map.

The framework and its application are demonstrated in details on the example of a habitat suitability study for an endangered herbaceous perennial plant called Wenatchee Mountains Checkermallow (*Sidalcea oregana var. calva*) in Chelan County, Washington, U.S. (Fish and Wildlife Service, 2001). In this case study, the uncertainty of inputs is represented by the variability of criteria weights associated with the essential habitat features of the plant. The practical objective of the case study is to delineate areas of high habitat suitability in order to maximize the potential area fit for population restoration. Given the uncertainty of weights assigned to habitat suitability criteria, the habitat suitability maps are accompanied by mean suitability and standard deviation maps obtained from multiple model executions. Subsequently, two different types of regions of interest are derived: regions of high suitability and low uncertainty (referred to as *robust areas*), and regions of high suitability and high uncertainty (referred to as *candidate areas*). Identifying the candidate areas is equally important to delineating the robust areas, because all areas of high suitability should be considered in order to maximize the total suitable area of future habitat. The identification of uncertainty sources in candidate areas is facilitated by the S-maps and ST-maps described in details in the next section.

The remainder of the paper is subdivided into the following parts. Section two provides the mathematical and algorithmic details of the iUSA framework. Section three describes the case study, focusing on data selection, geoprocessing, and simulation setup. Section four discusses the results of the simulations, first by reporting the outcomes of uncertainty analysis, followed by the description of sensitivity analysis results and the interpretation of combined iUSA results. Section five concludes the paper by summarizing the research and outlining future research directions.

2. iUSA Framework

2.1 Overview

The method reported herein consists of three stages (Figure 1, top). First, Monte Carlo (MC) simulations are executed to sweep through the uncertain input parameter space, where input values are sampled from probability density functions (PDFs). Criteria weights usually constitute the most subjective component of any multiple criteria decision analysis, significantly affecting its results (Malczewski, 1999, Hämäläinen and Salo, 1997). Consequently, in the application presented in section three, the input uncertainty is limited to criteria weights, which express preferences regarding the relative importance of habitat suitability criteria. Second, the uncertainty analysis is performed. The MC simulations produce multiple output suitability maps, which are summarized by calculating average suitability and uncertainty maps represented by the standard deviation of the mean suitability surfaces. Third, sensitivity analysis is employed in the form of the model-independent method of output variance decomposition, in which the variability of suitability maps is apportioned to every criterion weight, generating one first-order (S) and one total-effect (ST) sensitivity index map per criterion weight.

2.2 Sampling and Simulation

The specific algorithmic procedure, presented in Figure 1 (bottom), follows the method proposed in Saltelli et. al. (2010), referred to as quasi-random radial sampling. In the first step of radial sampling, two independent lists of N weight samples are generated based on predefined PDFs of the k criteria weights, referred to as sample lists N_A and N_B (Saltelli, 2002). Weight samples are produced using the quasi-random Sobol' experimental design (Sobol', 1993, Saltelli, 2002), which proved to be the most efficient when approximating the values of sensitivity indices (Saltelli et al., 2010). Using the N_A and N_B lists, radial samples are derived to perform a total of R model runs (Saltelli et al., 2010):

$$R = (k + 2)N \tag{1}$$

A radial weight sample is a sample Ab, in which the value of i-th weight a in sample j_A $(j_A \in N_A)$ is substituted with a value b from an equivalent sample in j_B ($j_B \in N_B$). For example, for the weight sample:

$$\left[a_{j1}, a_{j2}, a_{j3}, \dots, \boldsymbol{a_{ji}}, \dots, a_{jk}\right] \tag{2}$$

k radial samples *Ab* are generated in the form:

$$[a_{j1}, a_{j2}, a_{j3}, \dots, \boldsymbol{b}_{ji}, \dots, a_{jk}] \tag{3}$$

As a result, the model produces *R* suitability maps that are jointly called an R-stack (Figure 1, bottom).

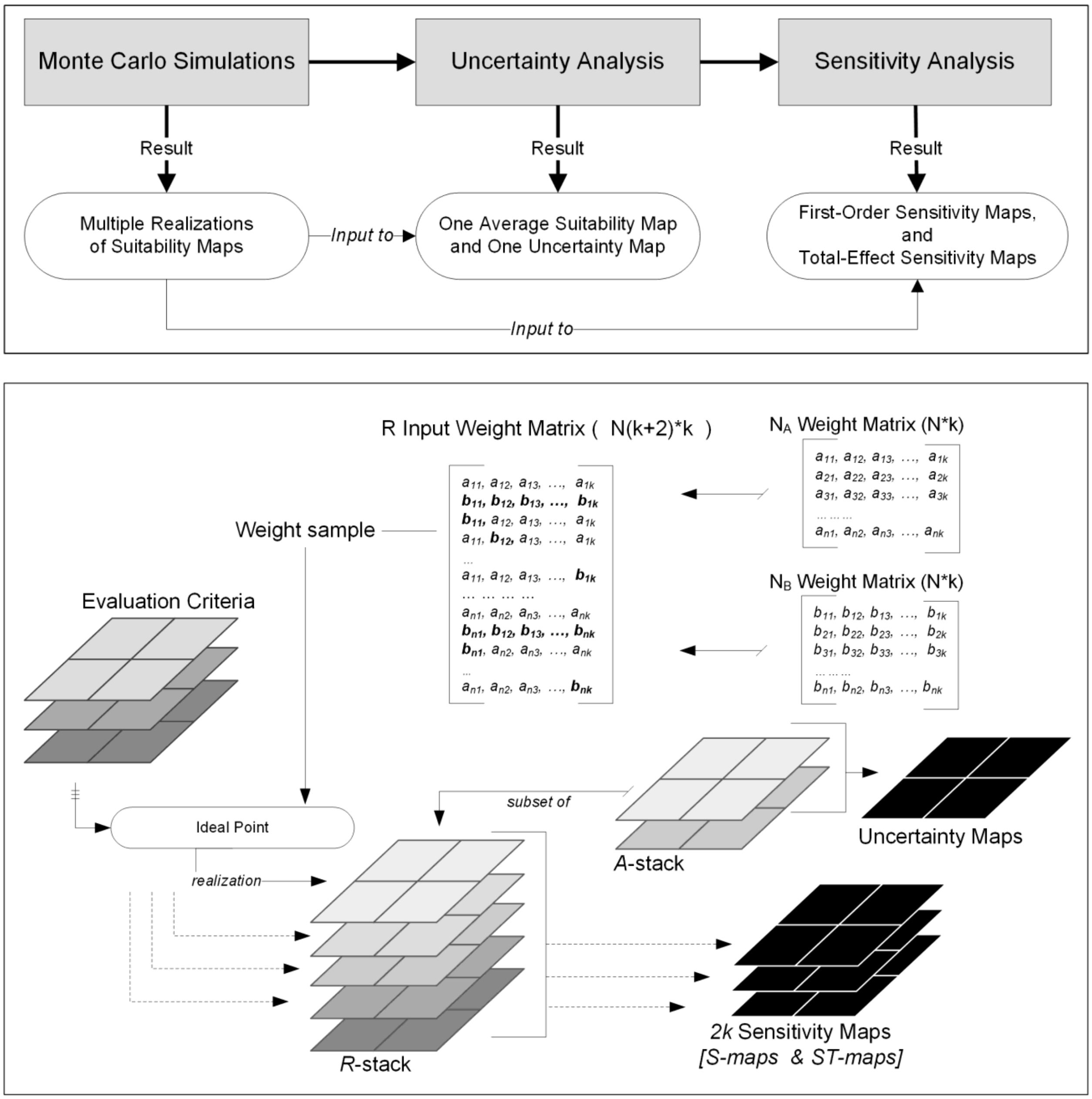


Figure 1 Procedure for integrated Spatial Uncertainty and Sensitivity Analysis of Suitability Weights (top: overview, bottom: detailed workflow).

To calculate each suitability surface, this paper employs the Ideal Point (IP) aggregation function (Hwang and Yoon 1981, Ligmann-Zielinska and Jankowski 2012). The IP function (Nyerges and Jankowski, 2009, Hwang and Yoon, 1981) calculates the final suitability score for every raster pixel (or other spatial unit) based on the separation of weighted multicriteria pixel score from two multicriteria reference metrics called the ideal point and the nadir, respectively. The ideal point represents a hypothetical pixel characterized by the most favorable values for the evaluation criteria considered in a given decision situation. The pixel that is closest to the ideal and, at the same time, farthest from the nadir point (i.e. a hypothetical pixel characterized by the worst outcomes for the evaluation criteria) is assigned the highest suitability score. In particular, assuming *k* standardized suitability criteria scores for pixel p_{xy}, where *xy* refers to *x* and *y* coordinates:

$$[p_{xy1}, p_{xy2}, p_{xy3}, \dots, p_{xyi}, \dots, p_{xyk}] \quad (4)$$

Calculate weighted standardized criterion scores v_{xy}:

$$[v_{xy1}, v_{xy2}, v_{xy3}, \dots, v_{xyi}, \dots, v_{xyk}] \quad (5)$$

where :

$$v_{xyi} = a_i^r * p_{xyi} \quad (6)$$

a_i is a weight assigned to criterion *i*, and $r \in N_R$ where N_R is a total set of *R* radial weight samples.

Next, identify the ideal (B*):

$$B^* = [v_1^*, v_2^*, v_3^*, \dots, v_i^*, \dots, v_k^*] \quad (7)$$

$$v_k^* = (max\ v_{xy}^k) |\ v_{xy} \in\ S \quad (8)$$

where *S* is the set of all pixels in a given weighted raster *k*.

Identify the nadir (W~):

$$W^\sim = [v_1^\sim, v_2^\sim, v_3^\sim, \dots, v_i^\sim, \dots, v_k^\sim] \quad (9)$$

$$v_k^\sim = (min\ v_{xy}^k) |\ v_{xy} \in\ S \quad (10)$$

Next, calculate separation measures (*Sep*) from B* and W~ for every $v_{xy} \in S$:

$$Sep_{xy}^* = \sqrt{\sum_{i=1}^{k} (v_{xyi} - v_i^*)^2} \quad (11)$$

$$Sep_{xy}^{\sim} = \sqrt{\sum_{i=1}^{k} \left(v_{xyi} - v_i^{\sim}\right)^2} \tag{12}$$

The final suitability score (C_{xy}) for every $v_{xy} \in S$ is computed as follows:

$$C_{xy} = \frac{Sep_{Xy}^{\sim}}{Sep_{Xy}^{*} + Sep_{Xy}^{\sim}} \tag{13}$$

Since the C_{xy} values are not affected by dependencies among the evaluation criteria (Hwang and Yoon, 1981), the IP is an attractive aggregation function for spatial evaluation problems characterized by a high likelihood of dependencies among the input layers.

2.3 Uncertainty Analysis

Given that the sample lists (N_A or N_B) are independent and the use of either of the lists leads to a relatively large number of model realizations, it is sufficient to use only one of the lists in order to compute a full range of the model response. Consequently, an A-stack of output sensitivity surfaces is selected from the R-stack (Figure 1). It consists of suitability maps calculated for all weight vectors in N_A. The A-stack can be further summarized by calculating (using local map algebra operations) its minimum (MIN), maximum (MAX), average (AVG), and standard deviation (STD) surfaces. At a minimum, the AVG and STD surfaces should be used, where STD is the uncertainty surface. This conjoint use of AVG and STD is reminiscent of the manner, by which geostatistical methods produce both an interpolated map and an associated error map accompanying the interpolated surface (Oliver and Webster, 1990, Krivoruchko, 2011, Zhang and Goodchild, 2002). The method thereby offers the means of accounting for and visualizing the spatial distribution of uncertainty for a given suitability map. AVG and STD surfaces allow for identification of the critical regions of suitability as presented in Table 1. Of particular interest is the quadrant one indicating the robust regions, and the quadrant two indicating the candidate regions.

	Low STD	High STD
High AVG	(1) High confidence priority regions	(2) Low confidence priority regions
Low AVG	(3) High confidence discard regions	(4) Low confidence discard regions

Table 1 Aspects of *robust* and *candidate* suitability regions. Robust: quadrant (1), candidate: quadrant (2)

The two extreme value maps of MIN and MAX offer additional information about the stability of suitability scores. The MIN map returns the minimum value of surfaces calculated on a cell-by-cell basis, revealing regions of repeatedly high values i.e. the *highs surface*. Specifically, it delineates areas that always score high relative to other sites. The MAX map returns the maximum value of layers calculated on a cell-by-cell basis. It is referred to as the *lows surface* because, for a given set of maps, it shows areas that always score low relative to other sites.

2.4 Sensitivity Analysis

Following the radial sample procedure (Saltelli et al., 2010) the R-stack is used to derive 2*k* sensitivity maps i.e., *k* number of S-maps and *k* number of ST-maps. In the particular context reported in this paper, the goal of SA is to find the criterion weights that have the most influence on site suitability. S is defined as a fractional, first-order (linear) contribution of a given criterion weight to the variance of the suitability scores calculated for a given pixel. The analyst uses S to look for influential criterion weights that, if fixed independently, would reduce the variance of the suitability the most. This is to say, weights with relatively high S values have the most impact on the variability of site suitability. There exists no established threshold for the S values. Values for all criterion weights have to be analyzed and their importance is determined based on their relative magnitudes (Saltelli et al., 2004). Calculating a complement to one of the sum of S values ($1 - \Sigma S$) allows for evaluating the interaction effects among inputs, which can be further described using the ST indices. ST finds the overall contribution of a given weight including its interactions with other weights (hence, indirectly, other suitability criteria).

The spatially-explicit iUSA framework requires the output variance decomposition for every spatial unit (e.g. pixel) of the suitability map, producing a separate sensitivity map for every input weight. Specifically, variance in suitability scores and the corresponding sensitivity indices are concurrently calculated for each and every pixel in the area of interest. For detailed definitions and formulas of the S and ST sensitivity indices the reader is referred to Saltelli et al. (2008), Lilburne and Tarantola (2009), Crosetto and Tarantola (2001), and Nossent et. al. (2011) among others.

3. Demonstration study

Habitat suitability analysis can be used as a method in ecosystem management to systematically screen land areas for introducing/reintroducing a particular species. The method produces a composite index of location suitability. The index demonstrates the usefulness of the site to meet the key life requisites for a given species. In this paper, the species of interest is the Wenatchee Mountains Checkermallow - *Sidalcea oregana var. calva* - referred to as Checkermallow in the following sections (Zimmerman and Reichard, 2005, Center for Plant Conservation, 2010). It is an endangered herbaceous perennial found only in six site locations in the Wenatchee Mountain range of Chelan County, Washington, U.S. with a total estimated population of 3,600 in 2001 (Fish and Wildlife Service, 2001) - Figure 2. Over 6,000 acres were designated for this plant as a critical habitat and the largest contiguous area that it is currently known to contain the species is the Camas Meadows Natural Area Preserve in Chelan County. Given the small size of the population and its scarcity, it is important to identify locations of suitable habitat for Checkermallow to promote its persistence and spread. Specifically, the goal of this study is to find the maximally suitable locations for Checkermallow communities by generating composite raster habitat suitability maps. This information will be useful for prioritizing sites that should be checked in the field for the existence of Checkermallow populations (most of the known locations were last checked in the late 1990s), and for selecting Checkermallow conservation zones (population hot spots). Essential habitat characteristics for the conservation of the species include moist meadows, saturated silt loams and clay loam soils especially in spring and early summer, the vicinity of open conifer forests dominated by Ponderosa pine and Douglas-fir, and mid-elevations (488 to 1000 meters). Because this population is so small, it is also at risk of extinction from random events, such as wildfires.

The extent of the study area is 66 square km with elevations ranging from 377 to 1377 meters. About 71% of the study area is forested, 10% is shrub land, and only 2% is wetland. Although there are no large water bodies present in the study area, creeks and small rivers are prevalent with a maximum distance of 900 m to the nearest stream.

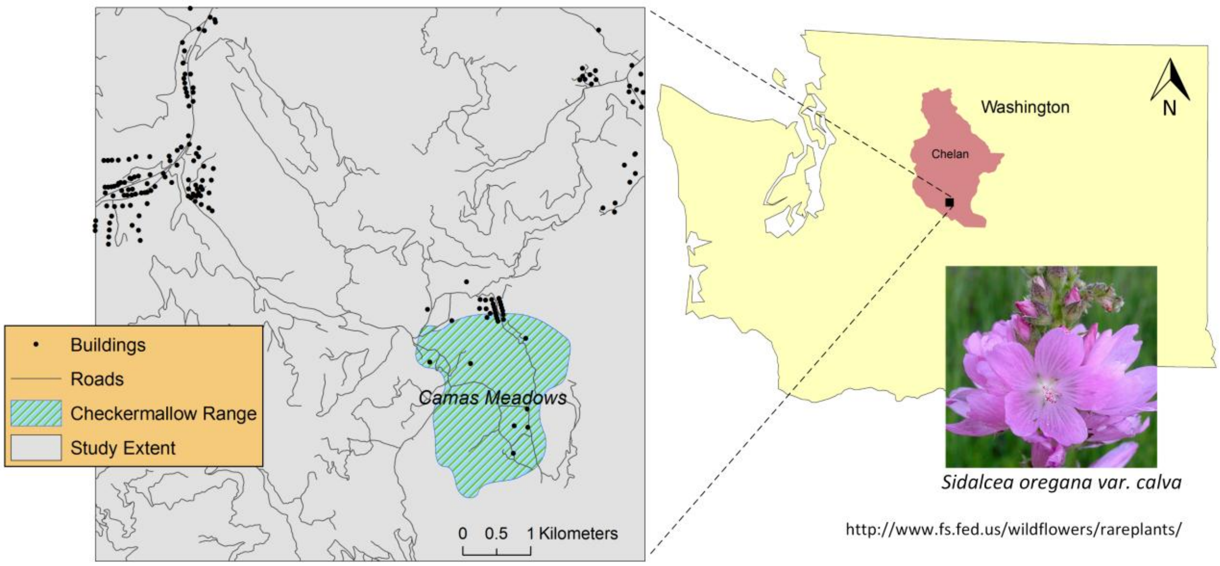


Figure 2. Study site; Chelan County, Washington, U.S. Camas Meadows Natural Area Preserve is the area of recorded plant communities. Data source: Washington Department of Natural Resources, Rare Plants and High Quality Ecosystems.

3.1 Habitat Criteria Selection and Geoprocessing

Based on the above description of the habitat characteristics suitable for Checkermallow, we used seven factors in the suitability analysis. Criteria surface names and the basic metadata are provided in Table 2. The extent of the study area was partitioned into raster cells with the resolution of 30m, resulting in 73170 cells (270 columns, 271 rows).

Map name	Date	Source	Description
WETSOIL[1]	2002	USDA-SSURGO	Saturated soils characteristic for wetlands (soils)
ELEV[1]	2009	USGS-NED	Elevation of plant occurrence (topography)
RADIATION[1]	2009	USGS-NED	Exposure to sunlight in spring (climate)
RAINFALL[1]	1971-2000	PRISM	Average annual precipitation (climate)
DEVDIST[2]	2000, 2012	CCPA and WA-DNR	Distance to rural residential development and transportation (land use)
STREAMS[1]	2011	USGS-NHD	Density of streams (hydrology)
CANOPY[1]	2001	USGS-NLCD-TCL	Density of coniferous forest (other plan communities)

Table 2 Selected Habitat Criteria. [1] Physical and biological habitat features, [2] Anthropogenic Threats. Sources: United States Department of Agriculture Soil Survey Geographic Database (USDA-SSURGO), United States Geological Survey National Elevation Dataset (USGS-NED), The PRISM Climate Group at Oregon State University (PRISM), Chelan County Planning and Administration Office (CCPA), Washington State Department of Natural Resources (WA-DNR), United States Geological Survey National Hydrography Dataset (USGS-NHD), United States Geological Survey National Land Cover Database Tree Canopy Layer (USGS-NLCD-TCL). Source data for ELEV, RADIATION, and CANOPY all share 30m resolution. The inputs to WETSOIL, STREAMS, and DEVDIST rasters are provided in vector format with positional accuracy of approximately 40m. PRISM, the most accurate climate data for the region, was downscaled from 4km to 30m (ArcGIS™ Resample Tool).

The soils dataset already includes an attribute that classifies soils based on their suitability for wildlife wetland plants requiring saturated soils. Since Checkermallow grows within the range of 488 to 1000 m.a.s.l., we assume that the midpoint of 744m is the most suitable elevation for the plant, and that all values above or below the midpoint diminish in their suitability. The elevation dataset was further used to calculate global solar radiation in the spring of 2011 (Watt Hours per sq meter). Using standard focal map algebra operations, we smoothed the source precipitation layer to fit the study data resolution of 30m.

Although fire hazard was mentioned as one of the major anthropogenic threats, we excluded this layer from the analysis due to its lack of variability within the area of interest (over 80% of area is within the fire hazard zone). Instead, distance to roads and distance to buildings were jointly used to represent the threats to the long-term existence of the plant. We also used stream density to represent the areas adjacent to or within the seepage zones. Finally, we calculated the density of forest using the available forest canopy land cover data.

All layers were tested for potential correlation. The pairwise Pearson correlation coefficient range was: $-.31 < r < .27$ ($p < .01$). Consequently, we assumed that the decision criteria were not linearly cross-correlated.

3.2 Criteria Standardization and Valuation

Six out of seven criteria used in the analysis are measured on different measurement scales (criteria six and seven in Table 2 are measured on the same density scale). In order to address the problem of incommensurate measurement scales, the criteria and their raw values need to be transformed into a common scale using a criterion standardization procedure. To maintain proportionality between the raw and standardized values, we used the ratio linear transformation for all layers except DEVDIST. Due to the nature of this criterion (distance), the score range transformation was used instead (Malczewski, 1999). Table 3 lists the valuations applied to each criterion. The final input criteria maps are depicted in Figure 3.

Name	Valuation	Justification/Preference is given to:
WETSOIL	B	Soils with higher suitability for wetland plants
ELEV	B	Sites closer to the midpoint elevations
RADIATION	B	More radiated areas
RAINFALL	B	Areas with higher precipitation levels
DEVDIST	C	It is a threat to the plant that should be avoided
STREAMS	B	Areas with higher potential for seepage
CANOPY	B	The plant is more common in the vicinity of conifer trees

Table 3 Criteria Valuation. Benefit (B) where higher value is considered preferable, and Cost (C) where lower value is preferable.

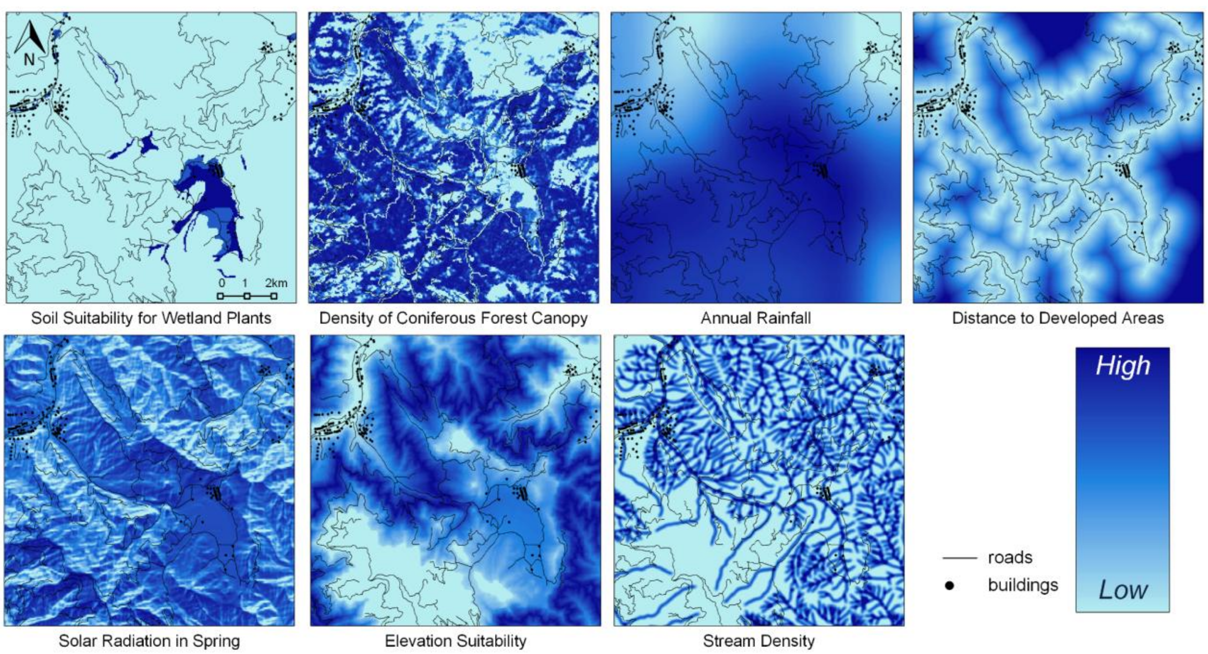


Figure 3 Standardized Checkermallow habitat factors used in the suitability analysis

3.3 Simulation Setup

Due to the lack of data on the relative importance of the habitat criteria, we assumed random uniform PDFs, with the range [0.0, 1.0] where every element has the same probability of being selected. All weights were derived independently from the PDFs, and were further recalculated to add up to 1.0.

As mentioned in section 2.2., we applied Sobol's experimental design with N=15360 weight samples used in the base runs for UA, resulting in 138 240 radial samples for SA (15360*(7+2)). The large value of N was dictated by the approximation error that occurred for lower sample sizes. Each S value should be within the 0 to 1 range so that the sum of S is less than or equal to 1. If large negative values for S are obtained, N is too small rendering an inadequate approximation. With the N=15360, negative values for the S index did not exceed 1.5% of all pixels, resulting in reliable sensitivity maps.

Given that the study site comprises 73 170 cells, we needed to compute over 10 billion (10^9) suitability scores (one suitability score for each of 138 240 samples of weights computed for each cell). For every pixel, we calculated unconditional variance of average score as well as approximated the sensitivity indices using the procedure described in section 2. The suitability calculations were run using the computing resources in the High Performance Computer Center at Michigan State University (http://icer.msu.edu/).

4. Results and Discussion

4.1 Spatial uncertainty analysis

The results of MC simulations were summarized by calculating two summary suitability surfaces presented in Figure 4 (top). The left map depicts an average habitat suitability surface of Checkermallow, computed as the mean of all Monte Carlo runs. The computed suitability scores fall within the 11% to 66% interval of the normalized suitability score range (0% - 100%). The areas of high suitability follow, in general, the high values of the ELEV criterion (Figure 3, bottom). For the uniform PDFs case this is not surprising, given that these pixels also have relatively high values for all criteria except WETSOIL. Consequently, the spatial distribution of high elevation values is accompanied by high values of other factors, resulting in a similar spatial pattern of output high habitat suitability.

If our land suitability analysis were to follow the current practice and the state of art in S-MCE, it would stop here. However, the AVG map provides an incomplete depiction of habitat suitability because, as seen in Figure 4 right hand-side (the STD surface), some of the high suitability sites are also characterized by a relatively high uncertainty associated with spatial distributions of suitability criteria. Hence, without the STD surface, the analyst could not have confidence that the high-scoring regions are robust, that is, stable under uncertain criteria weight values.

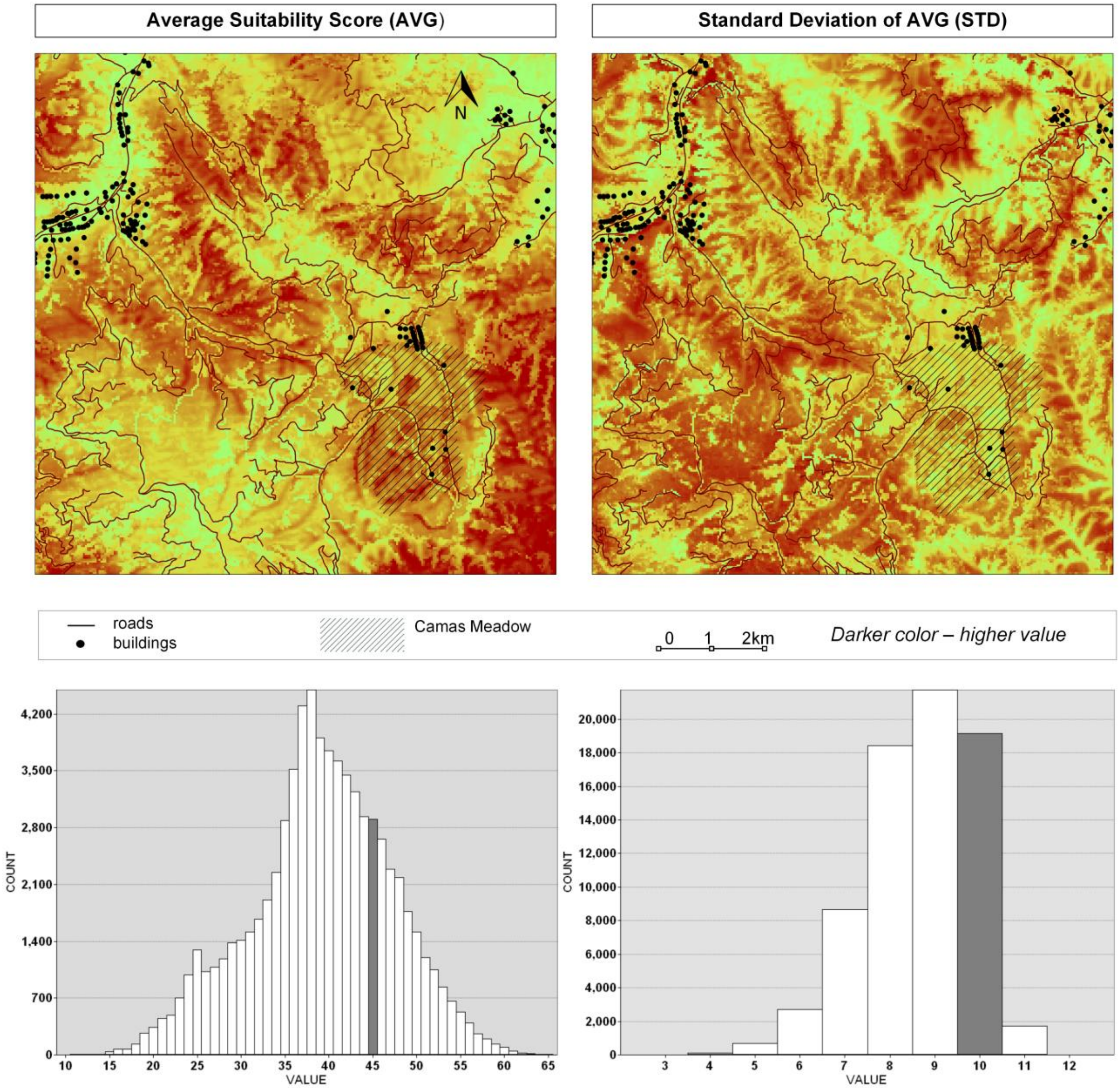


Figure 4 Average habitat suitability surface (AVG, top left) with its histogram (bottom left). Uncertainty surface (STD, top right) with its histogram (bottom right).

To further delineate the areas of interest, we assumed that the minimum AVG score for high suitability should equal 45% (Figure 4, bottom left). We also assumed that areas with STD over 10% are characterized by a relatively high suitability uncertainty (Figure 4, bottom right). As a result, we partitioned the surface into four regions (Figure 5 left): robust areas with AVG >= 45% and STD < 10% (referred to as High-Low or HL), candidate areas with AVG >= 45% and STD >= 10% (called High-High or HH), and two less productive categories of AVG < 45% (with high confidence when STD is less than 10%, and lower confidence when STD >= 10%). Observe that 30% of the regions suitable for Checkermallow habitat restoration (AVG >= 45%) are accompanied by a relatively high uncertainty due to criteria weights (STD >= 10%). Only after analyzing the suitability and uncertainty maps in tandem, we can suggest that 70% of the high suitability areas are robust for Checkermallow restoration (Figure 5). This is because the HL regions have values of STD lower than 10%, indicating that the suitability scores in these areas do not diverge by more than 10% from their average suitability value of 0.45 or higher. Conversely, if we assume that STD >= 10% indicates areas of substantial uncertainty, the HH regions have less robust suitability scores. In other words, HH areas are potentially suitable for habitat restoration but need further study due to the uncertain suitability scores.

Recall that the objective of this study is to maximize the potential area suitable for Checkermallow population restoration. In this situation, the decision maker would be not only interested in the robust HL regions but also in the candidate HH regions. Four most promising candidate regions are shown in Figure 5, right hand-side. To identify which habitat criteria (represented through weights) are behind the relatively uncertain high suitability scores for these regions we need to perform sensitivity analysis.

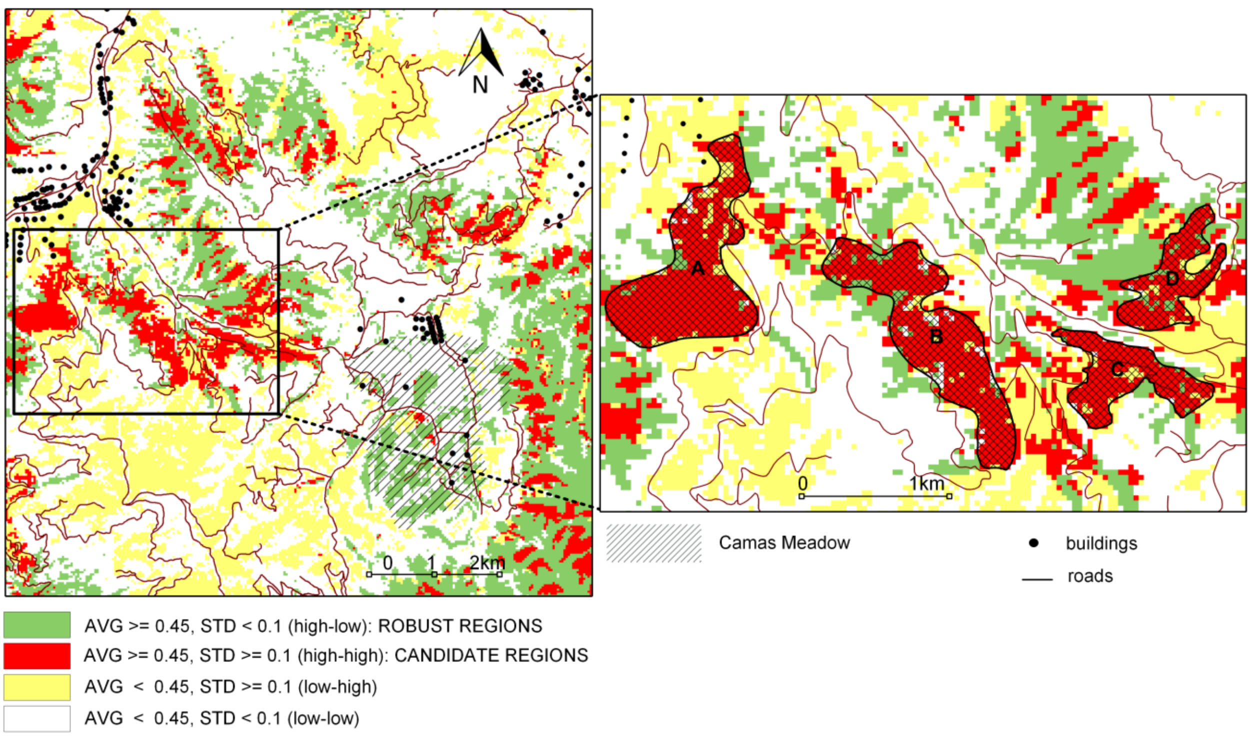


Figure 5 Left: habitat suitability classes (see table 1). Right: selected candidate patches.

4.2 Spatial sensitivity analysis

UA alone is of limited use if we want to determine the impact of individual criteria on shaping the uncertainty of suitability scores. Strictly speaking, the area of robust suitability cannot be increased without the identification of criterion weights contributing to the STD map. Specific locations and relative dominance of criterion weights influencing the uncertainty of suitability scores can be explored with the sensitivity maps, as shown in Figure 6. Since the spatial distribution of S-maps for the Checkermallow suitability criteria turned out to be very similar to the distribution of ST-maps, the following interpretation of results focuses only on the S-maps (we explain the relationship between the S and ST maps further in section 4.2.1).

The general pattern of weight sensitivities is spatially heterogeneous. However, the relationship between the input criteria (Figure 3) and their respective sensitivities (Figure 6) is quite complex. Three generalized 'input-sensitivity' groups can be identified. CANOPY (Pearson's $r = 0.62$, $p < .01$), ELEV ($r = 0.59$, $p < .01$), RADIATION ($r = 0.45$, $p < .01$), and RAINFALL ($r = 0.44$, $p < .01$) are characterized by positive spatial linear correlation, where the distribution of high values of input criteria is matched by the distribution of high values of their respective sensitivities. A converse negative correlation between inputs and the corresponding sensitivities can be observed for STREAMS ($r = -0.54$, $p < .01$). Finally, WETSOIL ($r = -0.13$, $p < .01$), and DEVDIST ($r = -0.23$, $p < .01$) have a mixed nonlinear relationship between the criteria and their S values, with some patches of high input values and low sensitivities, and other locations where high sensitivities correspond to high criteria values.

When analyzed conjunctively, the S-maps are quantitatively very different. In particular, if weight *k* has a high S value at a particular cell location, the other weights exhibit lower S values at the same location. This is not surprising given that every S-map renders a fractional contribution of a particular *k* to the total unconditional variance of the average suitability map. Hence, for every pixel, we can identify one *k* that has the highest S value, resulting in regions where this weight dominates other criteria weights. Hence, regions where Checkermallow occurrence is the most uncertain due to factor *k* are the regions of high S value for *k*.

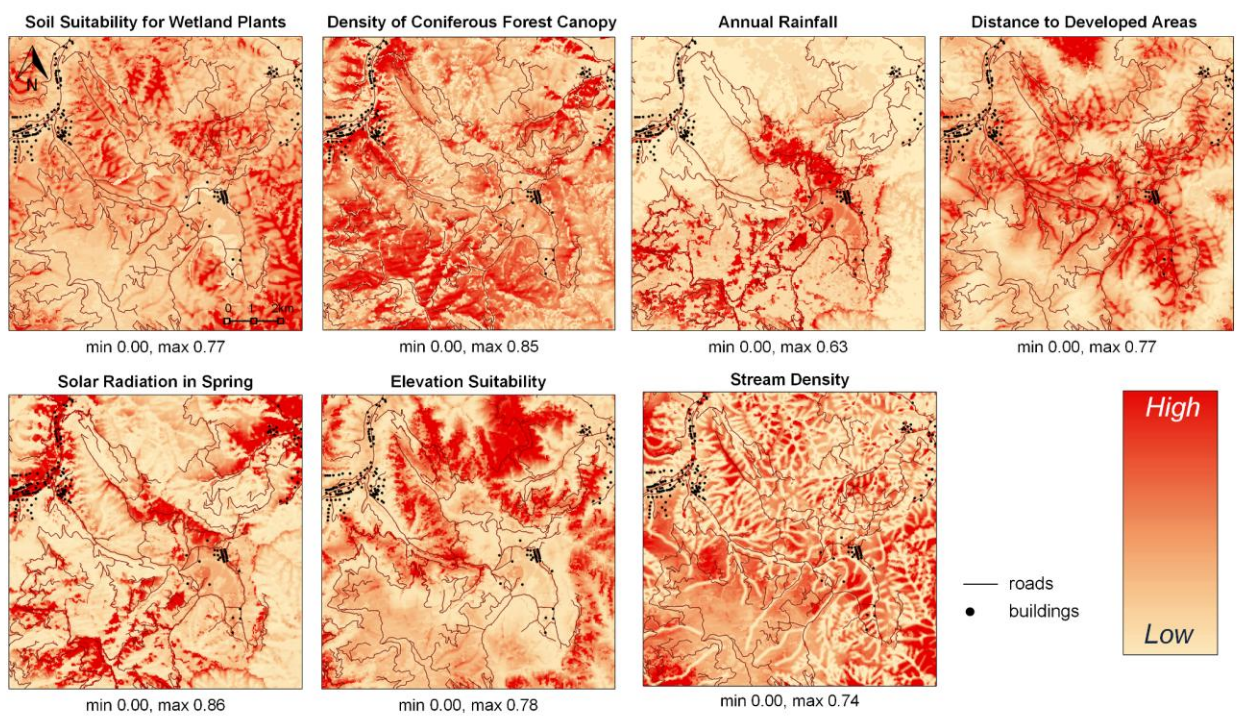
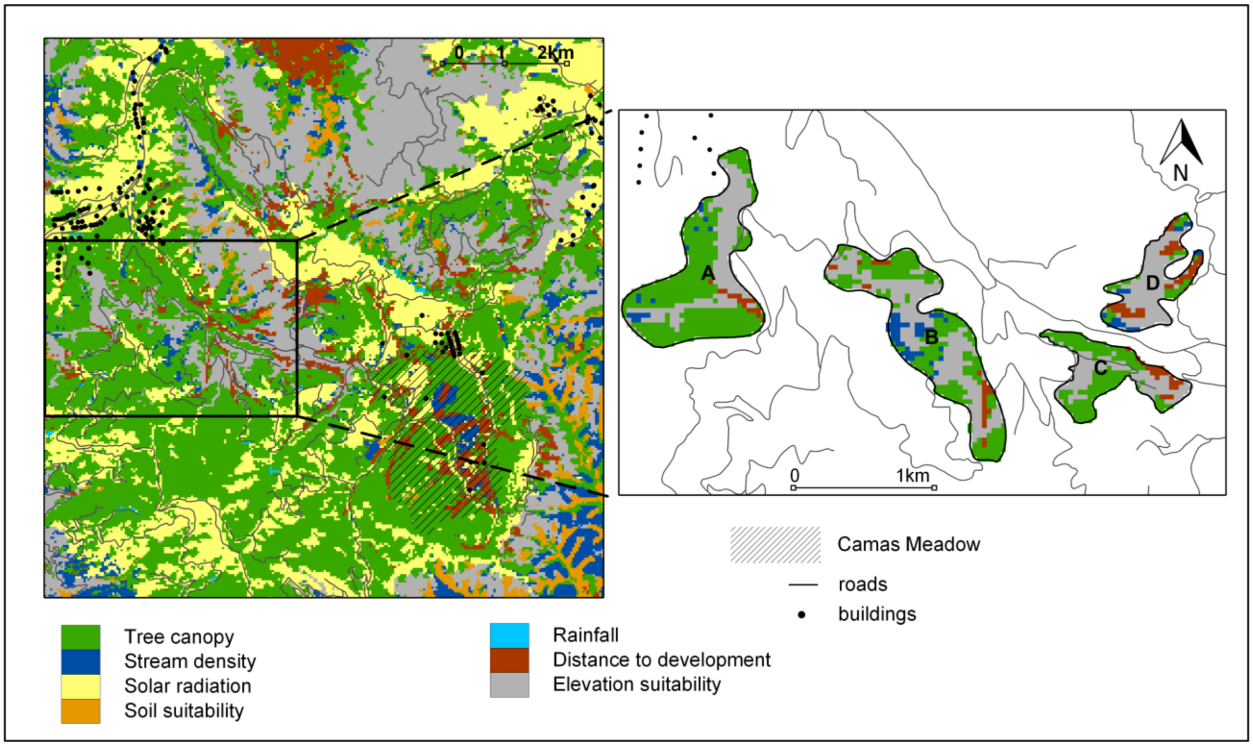


Figure 6. S-maps (first order sensitivity index maps) for the Checkermallow habitat suitability factors.

4.2.1 Maps of dominant sensitivities

We overlaid the S maps to determine, on a cell-by-cell basis, the criterion that has the maximum S sensitivity value. This procedure partitioned the space into regions of dominating weights, referred to as the weight dominance map shown in Figure 7 top left. For the entire study area the weights for three input criteria: CANOPY, RADIATION, and ELEV explain the vast majority of uncertainty associated with high suitability scores. For the selected candidate patches of higher uncertainty (Figure 7 top right), CANOPY and ELEVATION also proved to be the most influential. The other criteria: RADIATION, RAINFALL, and WETSOILS are not present in the weight dominance map of the four patches, suggesting that their influence on uncertainty in these areas is relatively low or even nonexistent.

Calculating a complement to one of the sum of S values ($1 - \Sigma S$) allows for evaluating the interaction effects among inputs, which can be further described using the ST indices. Consequently, to obtain a map of total interactions, we summed up the S-maps and subtracted the result from a homogenous raster with the value of one. A portion of this map is rendered in Figure 7 bottom left. It shows the pixels that fall within the candidate HH zone and, at the same time, have interaction effects ranging from 10% to 23%. Three observations can be made. First, the HH regions have, in general, a large area of interaction effects that are higher than the rest of the study area. Second, when compared with the S (non-interaction based) values of all criteria, the interaction effects are relatively low. Accordingly, the regions of high uncertainty accompanying high suitability could be explained by individual weights alone. Third (and correspondingly), the distribution of dominant ST values is very similar to the distribution of dominant S values in the four candidate patches (compare Figure 7 top right with the bottom right). Only 5% of the total area of the four patches differs in the dominant S versus the dominant ST values. Consequently, the analysis could be confined to the dominant S map. For both sensitivity indices, CANOPY is the dominating criterion weight shaping suitability uncertainty for patches A, B, and C, whereas ELEVATION prevails over all other criteria for uncertainty in patch D.



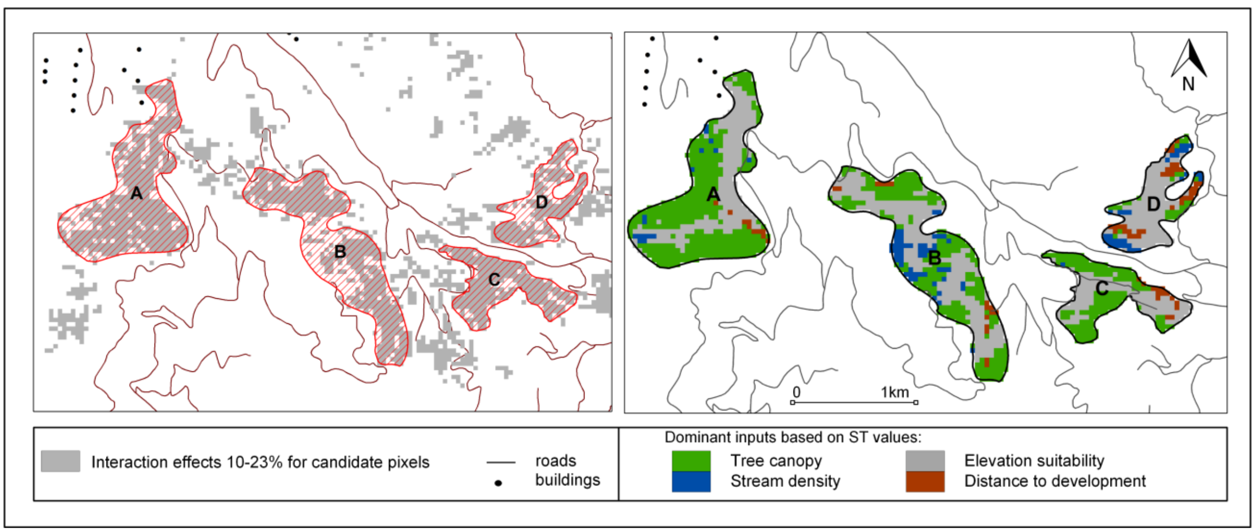


Figure 7 Top: regions of dominant weight sensitivities (left) and dominant criterion weight sensitivities for the four candidate patches (right). Bottom: interaction effects between criteria weights (left) and dominant weights in the candidate areas based on the total effect sensitivity indices (right).

4.3 Discussion

Quantification of habitat suitability for use in ecosystem management is a high cost endeavor that could rarely be done on a large scale. To address this problem, this paper introduces a comprehensive approach to suitability analysis resulting in a thorough diagnosis of habitat potential. The application of the approach provides information on regions of high suitability, their uncertainty, and the associated factors contributing to this uncertainty. Consequently, our final recommendation for Checkermallow habitat restoration is to focus on the high scoring regions depicted in Figure 5 left, which constitute about 26% of the study area. These regions of high average suitability can be further subdivided into two zones: [1] regions of low uncertainty constituting a stable suitability zone referred to as a robust habitat suitability region, and [2] regions of high uncertainty accompanying high suitability, associated with the variability of criteria weights. If selected, these regions should be further evaluated especially in relation to the weights associated with tree CANOPY and ELEVATION. Information obtained from the S and ST-maps (Figure 7) would be useful for further study that could focus on refining the role of the two criteria in determining Checkermallow habitat suitability. For tree canopy, a finer attribute resolution has a potential to reduce the uncertainty of the candidate regions. For example, the analyst could substitute the current CANOPY map with a vegetation map depicting two particular tree species associated with Checkermallow occurrence: Ponderosa pine and Douglas-fir. In addition, more accurate elucidation of criteria weights, performed by Checkermallow ecology experts, could improve the robustness of candidate locations.

Another practical implication of the S and ST-maps is that they help to uncover the spatial configuration of sensitivities. For example, factor reduction through fixing of the non-influential inputs to constant values, which is often performed as a result of variance-based SA, cannot be easily done for spatially heterogeneous inputs (Plata-Rocha et al., 2012). As shown in Figure 6, such inputs can render spatially variable sensitivities. Only if spatial variability is represented by low S and ST values, can we set a particular input to a constant. Moreover, the SA maps provide general information on spatial distribution of factor sensitivities. The analyst learns about inputs that cause high model outcome variability and, in addition, gains insight into the spatial structure of influential model inputs.

4.4 Limitations and future work

Unlike OAT (Chen et al., 2011, Chen et al., 2010b, Chen et al., 2013) the iUSA discussed herein has a high computational cost, which is its obvious limitation. In the demonstration study, we had to resort to supercomputing to generate the sensitivity maps. A potential solution to this problem is to employ linear regression (Manache and Melching, 2008), metamodeling (Marrel et al., 2011), or screening (Makler-Pick et al., 2011). An unexplored approach to reducing the computational cost is to calculate the sensitivity indices for a sample of locations, and derive the SA maps using interpolation instead of cell-by-cell S and ST calculations. If the decision maker is less interested in maximizing the area of suitable habitat and, instead, would like to focus on other ecological aspects like landscape configuration (the shape and connectivity of the suitability regions), the AVG map could be aggregated into selected landscape fragmentation statistics (McGarigal and Marks, 1995) and the standard non-spatial variance-based SA could be employed instead (Gómez-Delgado and Tarantola, 2006, Crosetto and Tarantola, 2001, Crosetto et al., 2000, Ligmann-Zielinska and Jankowski, 2010).

Another potential algorithmic improvement relates to the procedure of building dominant factor maps. The method used in section 4.2.1 relies on finding the maximum value among the S and ST indices for each pixel. This approach is inadequate when two or more factors have similar S or ST values. For example, if for a given pixel *p*: $S_{RADIATION}$ = 0.33, S_{CANOPY} = 0.31, $S_{STREAMS}$ = 0.32 then, based on the maximum value function, RADIATION would be rendered as the S value for *p* in Figure 7. However, CANOPY and STREAMS have S values roughly equal to RADIATION, and this fact should be included in the procedure of generating the dominant map. A more sophisticated cartographic algorithm is needed to address such cases.

In the simulated Checkermallow problem, the input variability was limited to criteria weights. Limiting the analysis only to uncertainty/sensitivity of weights works well enough for a demonstration study, but it would be too simplistic in real world ecosystem management situations. Such cases necessitate a more comprehensive iUSA where not only criteria weights but also criteria values are represented by probability density functions (Plata-Rocha et al., 2012). Lilburne and Tarantola (2009), for example, used multiple layers for each input map in their AquiferSim model in order to represent the potential variability of criteria values. In the Checkermallow case, a similar approach could be applied to the stream density layer, which uses a search distance to calculate the density. The distance could be set to different values generating different realizations of the stream density surface. Another component of the land suitability decision situation, introducing a potential uncertainty, is the aggregation function (like IP) used to derive the suitability surface. Studies suggest that the impact of function choice on the results can be significant (Makropoulos et al., 2008). Hence, the effect of using different aggregation functions could be also included in iUSA of habitat (land) suitability. In addition to analyzing the effect of potential variability in criteria values, the application of iUSA presented here could be extended by analyzing the effect of change in scale/spatial resolution of criteria values, providing that higher resolution data would be available.

5. Summary

This paper describes and demonstrates a comprehensive spatially-explicit uncertainty and sensitivity analysis approach for multicriteria land suitability evaluation. The proposed framework is based on the premise that various decision model factors are inherently uncertain and should be therefore evaluated using Monte Carlo simulations, where the variable input space is simultaneously perturbated, generating input parameter sets that are used to calculate multiple suitability maps. The results of such multiple multicriteria evaluations are summarized by computing: [1] an average suitability surface (AVG), [2] a standard deviation uncertainty surface (STD), and [3] a number of sensitivity surfaces.

As shown in the demonstration study, the AVG and STD maps allow for identification of critical regions of land suitability. Regions of high AVG and low STD signify robust suitability sites, whereas high AVG and high STD characterize candidate areas that are potentially suitable but need to be further investigated due to a significant level of uncertainty associated with the suitability scores. The latter can be explored using spatially-explicit variance-based sensitivity analysis, in which the spatial variability of habitat suitability and the associated uncertainty are decomposed and attributed to individual criteria weights.

Results obtained with the reported method can be valuable for land use manager or wildlife biologist in supporting their decision making concerning land use conversions, purchasing decisions, and re/introduction of species. The results help to identify highly suitable areas that are burdened by high uncertainty and then to investigate which specific factors contribute to the uncertainty. This information alone is valuable in helping to decide whether or not highly suitable but uncertain areas should be included in specific land use allocations.

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